Electric Grid State Estimators for Distribution Systems with Microgrids

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Abstract—In the development of smart grid, state estimation in distribution systems will likely face more challenges than that at the transmission level. This paper addresses one of such challenges, namely, the absence of topology information, by developing a forecast-aided topology change detection method and an event-triggered recursive Bayesian estimator to identify the correct topology. Simulation studies with microgrid induced changes are presented to illustrate the effectiveness of the proposed algorithm.

Index Terms—Distribution system state estimation, forecasting-aided state estimation, unscented Kalman filter, recursive Bayesian estimation, microgrid

1. INTRODUCTION

State estimation (SE) makes possible establishment of real-time models for the electric power grid by taking noisy measurements to estimate the states (e.g., voltage or current magnitudes and phase angles) at all the buses of the grid. SE has been traditionally a key function in the transmission systems and the resultant estimates are used for many functions in the operation and planning throughout the entire system, including power flow optimization and bad data analysis. However, to this date, utility companies have not done much in implementing SE in distribution systems.

Concerns for the environment and economy call for a large scale integration of distributed renewable sources and microgrids [1] into the distribution systems of the power grid. Such integration complicates the operation of distribution systems and will require enhanced distribution automation (DA). The system operators will therefore need more timely and reliable knowledge of the state of the system to properly monitor, control and economically dispatch power. Real-time distribution system state estimation (DSSE) will make control and management function effectively to help ensure the power grid’s reliability, efficiency and security [2]. DSSE has been established as a critical component of advanced DA, which is essential for operating and managing future power systems with distributed generators (DGs) [3]. In practice, DSSE is preferred to perform in predictive simulations to facilitate enhanced DA.

Unlike transmission system operators that can identify system topologies by the topology processor, distribution system operators usually lack specific topology information, e.g., DG connection status. However, such topology information is crucial for carrying out a meaningful DSSE. Wrong topology information can lead to divergence of the DSSE algorithm, or render wrong state estimates which may mislead system operators to take detrimental control actions.

As the power grid continues evolving into a smarter grid, utility companies will need to install (in stages) monitoring devices, similar to those mentioned in [4], on topology-changing components (e.g., switches) to achieve enhanced DA. This process will take a long time due to the very large scale of the distribution systems. In the case that control personnel cannot access the status of all (or some) of those switching devices due to limited (or missing/malfunctioning) infrastructure, it is necessary to develop DSSE algorithms that can autonomously detect topology changes and identify the correct system topology.

This paper begins with a comparative study on three classes of state estimators, namely, weighted least-squares (WLS) [5], extended Kalman filter (EKF) [6], and unscented Kalman filter (UKF) [7], with emphasis on DSSE. The WLS method obtains estimates of the states with a single snapshot of measurements. This static approach requires a significant amount of redundancy (which is difficult to achieve in the distribution system) to provide reliable estimates. The other two methods, i.e., EKF and UKF, have been developed primarily for forecast-aided state estimation (FASE) [6] which can provide predictive information. Such predictive information is very useful since it can be incorporated as pseudo measurements during the estimation process, resulting in increased measurement redundancy. It can also be integrated into other advanced distribution functions to provide predictive control.

We show in this paper, through a case study, that when the network topology is known (through, e.g., monitoring devices) a priori to the state estimators, UKF can yield better estimation accuracy than EKF and WLS. In the case that the topology is not known a priori, we develop autonomous DSSE algorithms which first utilize the predictive information to detect if there is a topology change. If there is none, the algorithm continues with the previous topology; otherwise, the algorithm employs a recursive Bayesian estimator to identify the correct model from multiple pre-defined topology models. The topology models are chosen according to, e.g., the connection status of switches. The recursive Bayesian estimator calculates the a posteriori probabilities for each model based on the difference between the real measurements and the estimated values. The resulting algorithm is referred to here as the event-triggered
recursive Bayesian estimator since the same topology model is retained until a change in a pre-defined condition on the state of the system triggers a re-identification of the topology. This approach is an extension to the multiple model selection idea developed in [8]–[10]. With the event-triggered recursive Bayesian estimator, the utilities will not need to install monitoring devices on all topology-changing components in one shot before they can obtain meaningful SE, thereby saving substantial cost and communication resources. Consequently, this method also makes DSSE algorithms more robust than conventional algorithms since an undetected topology change can easily make those algorithms diverge.

2. Mathematical Formulation

The conventional way to characterize the state of a power system is through the voltage magnitudes and phase angles at every bus. In an N-bus system, the state vector has the form \( x = [\theta_1, \theta_2, \ldots, \theta_n, |V_1|, \ldots, |V_n]|^T \), where \( \theta_n \) (\( n = 1, 2, \ldots, N \)) denote phase angles and \( |V_n| \) voltage magnitudes. Bus 1 is chosen as the reference bus, i.e., \( \theta_1 = 0 \). The measurement set \( z \) can be related to the state vector \( x \) by a nonlinear system model

\[
z = h(x) + n \tag{1}
\]

where \( h \) is a set of nonlinear functions, and \( n \) is the zero-mean Gaussian noise with covariance matrix \( \Sigma_n \). Due to low redundancy in real-time measurements at the distribution system level, the measurement vector \( z \) usually consists of very accurate virtual measurements, accurate real-time measurements, and less accurate pseudo measurements. The virtual measurements are the kind of information that does not require metering, such as zero injections and zero power flows at open switches. Real-time measurements are obtained from telemetered devices which are currently very limited in existing distribution systems. Pseudo measurements are estimated load injections which are subject to large errors.

A. WLS-Based Approach

The WLS-based approach minimizes the objective function

\[
\hat{x} = \arg \min_x [z - h(x)]^T \Sigma_n^{-1} [z - h(x)] \tag{2}
\]

The solution for \( x \) is obtained iteratively by linearizing (1) around the available estimate (at iteration \( j \)) and apply the Gauss-Newton algorithm to improve the estimate according to

\[
\hat{x}^{(j+1)} = \hat{x}^{(j)} + \left[ H^T(j) \Sigma_n^{-1} H(j) \right]^{-1} H^T(j) \Sigma_n^{-1} \left[ z - h(\hat{x}^{(j)}) \right],
\]

where the Jacobian matrix \( H(j) \) is formed from the first-order partial derivatives of \( h(x) \) with respect to \( x \) and evaluated at \( \hat{x}^{(j)} \), i.e., \( H(j) = \left\{ \frac{\partial h(x)}{\partial x} \right\}_{x=\hat{x}^{(j)}} \).

B. EKF-Based Approach

The EKF-based approach is commonly formulated with the following dynamic model [11]

\[
x(k+1) = F(k)x(k) + g(k) + w(k) \tag{4}
\]

where, for time instant \( k \), \( F(k) \in \mathbb{R}^{(2N-1) \times (2N-1)} \) is the state-transition matrix, vector \( g(k) \) is associated with the trend behavior of the state-trajectory, and \( w(k) \) is assumed to be zero-mean Gaussian noise with covariance matrix \( \Sigma_w \). Using (4) and the measurements arriving at instant \( k \), i.e., \( z(k) = h(x(k)] + n(k) \), the EKF formulation is given by

\[
\hat{x}(k+1) = \hat{x}(k+1) + \Sigma(k+1)H^T(k+1)
\cdot \Sigma_n^{-1} [z(k) - h(\hat{x}(k+1))] \tag{5}
\]

where \( \hat{x}(k+1) = F(k)\hat{x}(k) + g(k) \) and

\[
\Sigma(k+1) = \left[ H^T(k+1) \Sigma_n^{-1} H(k+1) + \Sigma_w^{-1} \right]^{-1} \tag{6}
\]

where \( H(k+1) \) is the measurement Jacobian evaluated at the predicted state vector \( \hat{x}(k+1) \) whose error covariance matrix is given by \( \Sigma_{\hat{x}}(k+1) = F(k) \Sigma_{\hat{x}}(k) F^T(k) + \Sigma_w \). In (4), \( F(k) \) and \( g(k) \) are usually obtained using the classic Holt-Winters method [11]. Note that the linearization of the system model in EKF leads to biased estimates and erroneous covariance. The following subsection describes UKF-based approach which is intended to solve this problem.

C. UKF-Based Approach

The UKF-based approach combines the unscented transformation (UT) with the traditional Kalman filter. The UT and the associated sigma points are defined in [7]. Using the state transition model (4) and system model (1), the UKF approach is formulated as [12].

\[
\hat{x}(k+1) = \hat{x}(k+1) + P_{xz}(k+1)
\cdot P_{zz}(k+1)^{-1} [z(k) - \psi_{x}(k+1)], \tag{7}
\]

where \( \hat{x}(k+1) = \sum_{i=1}^{2L+1} W_i^m \hat{x}(k+1) \), \( \psi_{x}(k+1) = \sum_{i=1}^{2L+1} W_i^m \hat{Z}^i(k+1) \), and

\[
P_{xz}(k+1) = \sum_{i=1}^{2L+1} W_i^m[(\hat{Z}^i(k+1) - \hat{x}(k+1))
\cdot (\hat{Z}^i(k+1) - \psi_{x}(k+1))^T] \tag{8}
\]

\[
P_{zz}(k+1) = \sum_{i=1}^{2L+1} W_i^m[(\hat{Z}^i(k+1) - \psi_{x}(k+1))^T + \Sigma_n]
\]

where \( i \) denotes the \( i \)-th column of the corresponding matrix, \( L \) is the number of the states to be estimated, \( 2L + 1 \) is the length of all sigma points set, \( W_i^m \) and \( W_i^c \) are the weighting factors for the associated means and covariance. \( \hat{x}(k+1) = F(k)\hat{x}(k) + g(k) \) and \( \hat{x}(k) \) is the sigma points set of \( \hat{x}(k) \). \( \hat{Z}^i(k+1) = h(\hat{x}(k+1)) \) and \( \hat{Z}^i(k) \) is the sigma points set of the predicted state vector \( \hat{x}(k+1) \) whose covariance matrix is given by

\[
\Sigma_{\hat{x}}(k+1) = \sum_{i=1}^{2L+1} W_i^m[(\hat{Z}^i(k+1) - \hat{x}(k+1))
\cdot (\hat{Z}^i(k+1) - \hat{x}(k+1))^T + \Sigma_w]. \tag{9}
\]
The covariance matrix of the state estimates $\hat{x}(k+1)$ is obtained as follows:

$$\Sigma_{\hat{x}(k+1)} = \Sigma_{\hat{x}(k+1)} - P_{XZ}(k+1)P_{ZZ}(k+1)^{-1}P_{XZ}(k+1)^T. \tag{10}$$

### 3. Event-Triggered Recursive Bayesian Estimator for Model Identification

When the network topology is not known a priori to the state estimators, the estimators need to determine the correct topology based on the limited number of real-time measurements. In this section, we develop an event-triggered recursive Bayesian estimator to detect topology changes and to identify the correct topology when necessary. Figure 1 is a flow chart depicting a forecast-aided state estimator using the proposed event-triggered recursive Bayesian estimator for model identification. Specifically, at time instant $k + 1$, the state estimator uses the predicted state information and the real-time measurements to decide whether or not a topology change has occurred with an innovation check. If there is not sufficient innovation to trigger an update on the topology model, the state estimator continues with the same model as at time instant $k$. Otherwise, a detected topology change will trigger the recursive Bayesian estimator to identify the correct topology model, which will be applied in the DSSE process. The resultant state estimates are used subsequently for the real-time measurements to decide whether or not a topology change has occurred.

### A. Topology Change Detection

Denote the measurement vector at time instant $k$ by $z^k = [z^k_1, z^k_2, z^k_3]$, where $z^k_i$ represents those measurements obtained from monitoring devices which are common to all topology models, $z^k_p$ and $z^k_v$ are, respectively, pseudo measurements and virtual measurements that are topology-specific and that can be used to distinguish different topology models.

At time instant $k + 1$, the forecasted state vector $\tilde{x}(k+1)$ as specified in (8) is used to calculate the forecasted measurement vector with

$$\tilde{z}^{k+1} = h(\tilde{x}^{k+1}). \tag{11}$$

We denote by $\tilde{z}^{k+1}_i$ the sub-vector in $\tilde{z}^{k+1}$ corresponding to the real-time measurements. Denote by $\nu^{k+1}$ the innovation vector corresponding to the real-time measurements at time instant $k + 1$, namely,

$$\nu^{k+1} = z^k - \tilde{z}^{k+1}. \tag{12}$$

The covariance matrix of $\nu^{k+1}$, denoted by $\Sigma_{\nu^{k+1}}$, is specified as the sub-matrix in $P_{ZZ}(k+1)$ given in (8) corresponding to the real-time measurements. We then define the $i$-th normalized innovation as

$$\nu^{k+1}_i = \frac{\nu^{k+1}(i)}{\sqrt{\Sigma_{\nu^{k+1}}(i,i)}}. \tag{13}$$

A topology change is detected if $\nu^{k+1}_i > \delta_i$ holds for at least one innovation. The threshold $\delta_i$ is determined offline based on historical operation data of the system.
the following, we will discuss how to set \( \delta_i \) to achieve good detection performance.

Since all measurements are subject to noise that are generally characterized by probabilistic models, we need to consider the possibilities of making wrong decisions, namely, the probability of false alarm and that of missing. We can assume that the normalized innovation \( v_{N,k+1}^{i}(i) \) is approximately Gaussian [15]. Specifically, if the model matches with the true topology, \( v_{N,k+1}^{i}(i) \sim N(0,1) \); if the model mismatches the true topology, \( v_{N,k+1}^{i}(i) \sim N(\mu_i,1) \) \((|\mu_i| > 0)\), where \( N(\mu,\sigma^2) \) represents a Gaussian distribution with mean \( \mu \) and variance \( \sigma^2 \). In our case here, we set the value for the mean \( \mu_i \) offline based on empirical data. Assuming the innovations are independent, we can evaluate the probability of false alarm by

\[
P_f = 1 - \prod_{i=1}^{N_I} \text{erf}(\frac{\delta_i}{\sqrt{2}}),
\]

where \( \text{erf}(\cdot) \) is the standard error function and \( N_I \) is the number of the innovations, and the probability of missing detection by

\[
P_m = \frac{1}{2} \prod_{i=1}^{N_I} \left[ 1 - \text{erf}(|\mu_i - \delta_i|) \right],
\]

where \( |\delta_i| \) should be set smaller than \( |\mu_i| \). To gain some insights into the values of these two probabilities, consider the following example. Let \( N_I = 3 \) and \( \mu_i = 5 \), which are reasonable numbers for a small scale network, we can see in Table I the corresponding probabilities of false alarm and missing detection for different values of \( \delta_i \).

<table>
<thead>
<tr>
<th>( \delta_i )</th>
<th>( P_f )</th>
<th>( P_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.1304</td>
<td>1.3475e-015</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0368</td>
<td>8.4244e-012</td>
</tr>
<tr>
<td>3</td>
<td>0.0081</td>
<td>1.2794e-008</td>
</tr>
</tbody>
</table>

In practice, we can use those two probabilities as a guideline to define \( \delta_i \). Ideally we would like to minimize both probabilities. However, \( P_f \) is a decreasing function of \( \delta_i \) while \( P_m \) is an increasing function of \( \delta_i \), thus a tradeoff needs to be found. In this paper, we do not seek to find the statistically optimal tradeoff solution. Fortunately, both scenarios usually do not have detrimental effects on the DSSE process. False alarm will trigger the topology identification procedure which will eventually find the true topology anyway. The probability of missing detection is always very small according to Table I. In practice, even if missing detection happens for an innovation check, topology errors could be detected in the residual check step shown in Fig. 1 or during the next time instant.

**B. Topology Identification**

In the case that a topology change is detected at time instant \( k+1 \), we search through a model bank to identify the correct network topology. We store all possible topology configurations of the system in the model bank, and perform SE for all topology models with the same set of real-time measurements. The probability of each model is calculated based on the SE results of all the models driven by the real-time measurements. Calculation of those probabilities is described as follows:

The *a posteriori* probability (APP) for any of the model is calculated using the recursive Bayesian estimation algorithm based on Monte Carlo simulations at time instant \( k+1 \). During the Monte Carlo simulations, \( z_{r,k+1}^{i} \) does not change while the errors in \( z_{r,k+1}^{i} \) and \( z_{r,k+1}^{i} \) are generated by a random number generator. Consider the APP for a particular topology model and for a particular realization of the error set. Using the Bayes’ rule, we can write the APP of model \( \eta_i \) as

\[
Pr(\eta_i|x^m) = \frac{Pr(e^m_i|\eta_i) Pr(\eta_i|x^{m-1})}{\sum_{j=1}^{N_m} Pr(e^m_i|\eta_j) Pr(\eta_j|x^{m-1})},
\]

where \( m \) is the iteration number of the Monte Carlo simulations, \( N_m \) is the total number of models, \( \eta_i \) is the \( i \)-th model, and \( \xi^m = \{e^m_1, e^m_2, \ldots, e^m_{N_m}\} \) is the set of error vectors of various models. The error is the difference between the estimated measurements and the real-time measurements, i.e., \( e^m = z_{r,k+1}^{i} - \hat{z}_{r,k+1}^{i,m} \). Let \( \hat{x}_{r,k+1}^{i,m} \) be the state estimates of the \( m \)-th iteration obtained from the WLS-based approach for the \( i \)-th model. \( \hat{x}_{r,k+1}^{i,m} \) is the part of \( \hat{x}_{r,k+1}^{i,m} \) corresponding to the real-time measurements. \( Pr(\eta_i|x^{m-1}) \) is the APP of model \( \eta_i \) in the previous iteration. The *a priori* probability \( Pr(e^m_i|\eta_i) \) can be calculated from Gaussian distribution, and equation (16) can be re-written as [8], [9].

\[
Pr(\eta_i|x^m) = \frac{\beta_i \exp(-\frac{1}{2} e^m_i T \Sigma^{-1}_{x_{r,i}^m} e^m_i) Pr(\eta_i|x^{m-1})}{\sum_{j=1}^{N_m} \beta_j \exp(-\frac{1}{2} e^m_j T \Sigma^{-1}_{x_{r,j}^m} e^m_j) Pr(\eta_j|x^{m-1})},
\]

where \( \beta_i = [\det(\Sigma_{e_i}^m)]^{-1/2} \) and the error covariance matrix \( \Sigma_{e_i}^m \) is defined as

\[
\Sigma_{e_i}^m = H_r(\hat{x}_{r}^m) [H(\hat{x}_{r}^m)T \Sigma_{x_{r}}^{-1} H(\hat{x}_{r}^m)]^{-1} H_r(\hat{x}_{r}^m)T + \Sigma_{n_r},
\]

where subscripts \( r \) denotes the Jacobian matrix (\( H_r \)) and the noise covariance (\( \Sigma_{n_r} \)) corresponding to the real-time measurement.

In the Monte Carlo simulation, all models are first initialized to be equally likely, i.e., \( Pr(\eta_i|x^0) = \frac{1}{N_m} \). At each iteration, new probabilities are updated from the probabilities computed at the previous iteration. Since \( z_{r,k+1}^{i,m} \) is the set of results driven by the current topology model following physical laws, it will drive the APP \( Pr(\eta_i|x^m) \) to converge to the largest probability as \( m \) increases if \( \eta_i \) is the correct model. In the end, the model with the highest probability is selected as the correct system topology. This will be discussed further in the next subsection.

The newly identified topology is henceforth applied from time instant \( k+1 \) in the SE process until a next topology change triggers another round of topology identification. When the new topology is applied, the previous state knowledge, e.g., the forecasted state estimates \( \hat{x}_{r,k+1}^{i,m} \), will not be reliable since
it was based on the old topology. The topology change may also cause fluctuations in the state estimates which may cause divergence. To circumvent this, we can scale the covariance matrix of the state estimates $\hat{x}(k+1)$ by dividing $\Sigma \approx \lambda^t$, by a forgetting factor $\lambda$, similar to the recursive least-squares algorithm [16].

$$\lambda_{t+1} = \lambda_t + (1 - \lambda_t)(1 - e^{-t/\Gamma}) \quad (0 < \lambda_t \leq 1),$$

(19)

where the subscript $t$ is reset to zero when a topology change is detected. The initial condition $\lambda_0$ is always set to be a small value to expedite convergence of the SE process. $\lambda_t$ increases exponentially with $t$, scaled by a pre-defined constant $\Gamma$.

C. Complexity and Performance

We now examine the convergence behavior of the proposed recursive Bayesian estimator, and show that the probability for selecting the correct model converges asymptotically to 1. For convenience, let $p_i^m = \Pr(\eta_1^m)$ and $\alpha_i = \beta_i \exp(-\frac{1}{2} \epsilon_j^T \Sigma^{-1}_{\eta} \epsilon_j)$. From (17), we have

$$p_i^m - p_i^{m-1} = \frac{p_i^{m-1}[(1 - p_i^{m-1})\alpha_i - \sum_{j \neq i} \alpha_j]}{\sum_{j=1}^n \alpha_j p_i^{m-1}}.$$  

(20)

Assume that the $i$-th model is the true topology, and all other models are very different from the true model. Since there is a mismatch between the real-time measurements and the $j$-th ($j \neq i$) model, $\epsilon_j^T \Sigma^{-1}_{\eta} \epsilon_j$ is most likely significantly larger than $\alpha_j$. As such, we can approximate (20) as

$$p_i^m - p_i^{m-1} \approx \frac{p_i^{m-1}[(1 - p_i^{m-1})\alpha_i]}{\sum_{j=1}^n \alpha_j p_i^{m-1}} > 0,$$

(21)

which shows that the APP for the true model increases exponentially with each iteration with a high probability. In the initial stage, the increase is very fast. As the value of APP approaches 1, the increase tapers off. To expedite the model identification process, we set a threshold to select a model whenever its APP value goes above a pre-defined threshold.

For any practical distribution system, the number of the normal topology models is naturally constrained, especially if fault scenarios are not included. Fault scenarios are not considered in this paper, for we assume that they are eliminated along with bad data [15] before topology change detection is performed. In fact, with the evolution of the smart grid, more and more critical topology-changing components will be monitored by measuring devices. This will steadily reduce the number of normal topology models and improve the efficiency of the recursive Bayesian estimator. Moreover, the model bank idea can be potentially integrated with distributed SE in which local estimators only deal with small portions of the entire system. In each small scale system, the topology change does not happen often, hence the topology identification procedure is usually not triggered in the local estimator, and that will save a lot of computational resources.

4. CASE STUDY

In this section, we compare the performance of three estimators on an IEEE 12-bus distribution system augmented with two microgrids as shown in Fig. 2. The two microgrids constitute two additional buses, i.e., Buses 13 and 14, that are connected, respectively, to Buses 6 and 12 through switches. Each microgrid can provide 10% of the original total load demand when connected to the main grid. The monitoring device installed on the feeder can measure voltage magnitude as well as real and reactive power flows. The first bus is used as the reference bus in our simulation. The voltage phase angle of an interested bus is calculated as the deviation of that bus’ actual phase angle from the reference bus’ phase angle. The unit for the voltage magnitude is per-unit (p.u.), and the unit for phase angle is degree.

The dynamics of the system are simulated by introducing a load increase of 0.05% on all load buses plus a small random jitter at each time instant. The random jitter is bounded such that sudden large variations in state variables are excluded. A load flow calculation is performed to update line flows, voltage magnitudes and phase angles throughout the system and to provide real-time and pseudo measurements which are corrupted by additive white Gaussian noise. The Gaussian distribution for the measurement noise is truncated at $\pm 3\sigma$ to avoid gross errors which should otherwise be eliminated by bad data analysis [15]. The standard deviations for the voltage magnitude and power flows are set to 0.1% (voltage) and 1% (power) of actual values. The virtual measurements like zero injections are considered to be very accurate with standard deviations of $10^{-4}$. The pseudo measurements are obtained from load estimates with standard deviations of 20% of their actual values. For the dynamic state equations, the value of $F(k)$ and $g(k)$ are obtained from the Holt-Winters method [11] with parameter values $\alpha = 0.8$ and $\beta = 0.5$. The value of the covariance matrix $\Sigma_{uv}$ is set to be $10^{-6}$.

This study focuses on the topology changes caused by the connection or disconnection of microgrids. The model bank contains model $\eta_1$: switch S1 closed and switch S2 closed, model $\eta_2$: switch S1 closed and switch S2 open, model $\eta_3$: switch S1 open and switch S2 closed, and model $\eta_4$: switch S1 open and switch S2 open. Other possible scenarios like activation of protective devices or close of normally open points are network-specific and can be dealt with in a similar fashion by adding these possible topology models into the model bank.

This study considers two scenarios when evaluating the per-
formance of the estimators. In the first scenario, the network topology is known (through, e.g., monitoring devices) a priori to the state estimators. In the second scenario, the topology is not known a priori to the estimators. We then evaluate the performance of the event-triggered recursive Bayesian estimator in identifying the correct topology of the network.

A. Topology Known a priori

When the topology is known a priori to the state estimators, we carry out Monte Carlo simulations to compare the performance of the three estimators. We define the performance index for the three estimators at all time instants as

$$\xi_k = \frac{\sum_{i=1}^{M} |\hat{z}_i(k) - z_T(i,k)|}{\sum_{i=1}^{M} |z_i(k) - z_T(i,k)|},$$

(22)

where $i$ is the Monte Carlo simulation iteration number, $M$ is the total number of iterations, $\hat{z}_i(k)$ is the estimated measurement set, $z_T(i,k)$ is the true measurement set with no noise, and $z_i(k)$ is the noisy measurement set.

Figure 3 shows the averaged performance indexes as defined in (22) for WLS-, EKF-, and UKF-based state estimators for 60 time instants. The topology of the studied system is configured as model $\eta_1$ for the first 29 time instants and model $\eta_2$ for the remaining time instants. The figure shows that UKF outperforms both EKF and WLS.

![Fig. 3. The performance indexes for UKF-, EKF-, and WLS-based state estimators](image)

B. Topology not Known a priori

When the topology is not known a priori, we employ the event-triggered recursive Bayesian estimator to detect topology change and to identify the correct topology. Figure 4 shows the probabilities of the four topology models progressing through 100 iterations. The probabilities are obtained by averaging the results of 100 independent simulations. The probability of the correct model increases with the number of iterations and is above a predefined threshold (0.5 in this study) when the algorithm stops at the 100-th iteration.

5. Conclusion

This paper considers DSSE for both the cases that the network topology is known and that it is unknown a priori to the state estimators. We investigate and compare three classes of state estimators in terms of computational complexity and performance. We also propose an event-triggered recursive Bayesian estimator for the case that the network topology is unknown a priori and show that it achieves very accurate model selection performance.

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