Scheduling Algorithms for PHEV Charging in Shared Parking Lots

Jing Huang, Vijay Gupta and Yih-Fang Huang

Abstract—As the penetration level of plug-in hybrid electric vehicles (PHEVs) increases, the stress on the distribution system will become more pronounced. We concentrate on PHEV charging in public infrastructure, such as shared parking lots in commercial office campuses and shopping malls. In such clustered scenarios, PHEV charging demand can vary significantly and stochastically with time, and the overall system performance is a trade-off between the peak load on the distribution system and the consumer specified deadlines that are met successfully. However, unlike distributed residential charging, this situation permits a centralized scheduler based control of the vehicles that are charged at any time. This paper considers the design of scheduling algorithms to optimize the system performance. We show that through proper design of the scheduling algorithms, the parking lot owner can balance distribution system load and quality of charging service.

I. INTRODUCTION

Electrification of the vehicle fleet offers a great potential to reduce both greenhouse gas emissions [1] and the daily transportation cost [2]. Moreover, the prospect also means increased revenue for both auto manufacturers and utility companies. Accordingly, there is a great interest among both policy makers and industrial players to promote a higher level of penetration of plug in hybrid electric vehicles and plug-in electric vehicles (denoted by PHEVs from hereon).

However, significant technical hurdles towards reaching that goal still remain. One of the most important challenge is that the distribution system of the electric grid may be severely stressed as the PHEV penetration level increases. High penetration of PHEVs (as new loads) will add significant electricity demand. More importantly, the introduction of these loads is highly correlated in both time (for instance, most PHEVs will be plugged in for charging after the commute from office in the evening) and space (for instance, in large office parking lots, many PHEVs will be clustered together for charging). Thus, even though the generation system may be able to supply enough energy to charge the PHEVs, the distribution system may still be stressed temporally and spatially [2]–[4] if the PHEVs are introduced as conventional loads and serviced in an uncoordinated manner (see also [5]–[7]).

Coordinated charging through scheduling the time at which different PHEVs are charged has been proposed to mitigate these issues [8], [13]. Many works have studied coordinated charging of PHEVs at consumer residences with various control strategies, such as time of usage (TOU) based charging [12], convex linear and quadratic programming [8], sequential quadratic optimization [3], [9], particle swarm optimization [10], non-cooperative games [11], and so on.

As opposed to these works, this paper concentrates on PHEV charging in public infrastructure such as shared parking lots in large office and university campuses, or in shopping malls. Sole reliance on residential charging imposes severe constraints on driving patterns and increases required vehicle depletion range, thus hindering the popularity of PHEVs. For example, assuming no charging infrastructures other than that at the owner’s residence, the required minimum vehicle depletion range is estimated to be 40 miles. Using alternative infrastructure, this can be lowered to 27 miles. This will lead to a reduction of over $8,000 in the cost of each PHEV [14]. Establishing public infrastructure for PHEV charging will be an effective way to persuade people to adopt PHEVs.

Coordination of charging in public infrastructure presents both unique opportunities and challenges. The charging demand in such locations is highly variable depending on the stochastic customer arrival process. An uncoordinated charging strategy may lead to a load profile with a high peak to average power level. On the other hand, imposing a strict peak power constraint may lead to many vehicle charging deadlines not being met. On the other hand, the coordination of the vehicle charging need not be done in a decentralized fashion, as in residential environments. Rather, a centralized controller which schedules the charging time of PHEVs can be designed to optimize the system performance, by considering both the load profile and the consumer demands. We design and analyze the performance of such scheduling algorithms. Note that, we do not consider the possibility of either smart charging through two-way communication or the usage of PHEVs as distributed storage devices and generation resources (also known as vehicle-to-grid (V2G)) in this paper, since both are considered to be unlikely in the near term.

Contributions: This paper develops and analyzes centralized scheduling strategies for PHEV charging at public infrastructure. Specifically, we consider coordinated PHEV charging for two models, namely large corporate parking plazas (that may provide PHEV charging to their employees as a benefit) and shopping mall parking plazas (that may provide PHEV charging as a promotion or attraction to potential customers). We introduce a stochastic model for the charging demand at these locations and formulate the problem of balancing charging demand with system load. We design and analyze the performance of many scheduling algorithms for PHEV charging at large parking plazas. In particular, for some cases, we are able to identify the optimal scheduling algorithm. Numerical examples illustrate the effectiveness of proper scheduling strategy on balancing the utilization level of power resources and customer perceived quality of service.

Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556. jhuang6, vgupta2, huang@nd.edu
II. PROBLEM SETTING

Consider the system architecture shown in Figure 1. The parking lot consists of multiple parking spaces, each equipped with a charging meter that the customers can plug their PHEVs into. Additionally, the consumers can enter information such as the departure deadline, desired amount of charging and the preferred charging rate. The parking lot owner, or the service provider, can design a centralized controller to schedule the charging of each PHEV. The controller has two performance metrics:

- Due to economical and hardware concerns, the number of PHEVs that can be charged simultaneously (i.e. the peak load) should be minimized.
- To satisfy the customer demand, the number of PHEVs that miss their charging deadline (i.e., are not fully charged to the level requested by the customer till the respective departure deadline) needs to be minimized.

Since the PHEV arrivals are stochastic, there is clearly a trade-off between the two performance metrics. A relevant optimization problem to design the controller can thus be to minimize the probability of a vehicle missing its charging deadline; however, the peak load can be very large. In particular, we consider two scenarios:

• In the office campus scenario, all PHEVs have the same deadline $T_2$ ($T_2 > T_1$) corresponding to the evening departure time for all employees.
• In the shopping mall scenario, we assume that the deadline for any particular PHEV is given relative to its arrival time. Specifically, the deadline is $\Delta T$ time after the PHEV arrival, where $\Delta T$ is uniformly distributed in the interval $(0, \Gamma)$. The deadlines for different PHEVs are mutually independent and they are also independent of the arrival time and the charging levels.

III. MAIN RESULTS

This section investigates the performance of various scheduling algorithms with a prescribed power profile in the system. We first consider a benchmark scenario where charging starts as soon as a PHEV is plugged in and continues till it is fully charged. Charging in this scenario is uncoordinated and the distribution system components do not impose any constraint on the number of PHEVs that can be charged simultaneously. Thus, this scenario does not miss any charging deadline; however, the peak load can be very large. Subsequently we evaluate various scheduling algorithms when there is a constraint on the number of PHEVs that can be charged simultaneously.

A. Vehicle Arrival Model

For the charging system, the vehicle arrival time refers to the time of plug-in to charge the PHEV. In real life, the arrival time is determined by the time when the PHEVs arrive at the charging station and hence is stochastic. Denote the total number of arrived PHEVs up to time $t$ by $N(t)$. We model the PHEV arrival process as a time-inhomogeneous Poisson process with a rate parameter (or intensity) $\lambda(t)$ [15]. The rate parameter can usually be determined from historical data or estimated from real time traffic data. In this paper, for simplicity, we assume that PHEVs arrive in time interval $[0, T_1]$ so that $\lambda(t)$ is defined as

$$
\lambda(t) = \begin{cases} 
\lambda & t \in [0, T_1] \\
0 & \text{otherwise.}
\end{cases}
$$

The Poisson model is widely used in many practical applications to model, e.g., bus passenger arrivals, supermarket consumer arrivals, etc.

B. Charging Pattern and Departure Model

This paper assumes that each PHEV charges at a constant rate $\psi$ kW until it is charged to the level desired by the customer. The aggregated load profile is thus the number of vehicles being charged at a particular time multiplied by the rate $\psi$. While multiple PHEV charging rates can be considered, this assumption simplifies the analysis and can serve as a benchmark. We assume that the required charging level for a PHEV at arrival is a sequence of independent and identically distribution (i.i.d.) random variables. The arrival times and required charging levels are assumed to be mutually independent.

The final component is the deadline by which the consumer needs the PHEV to be fully charged. In particular, we consider two scenarios:

- In the office campus scenario, all PHEVs have the same deadline $T_2$ ($T_2 > T_1$) corresponding to the evening departure time for all employees.
- In the shopping mall scenario, we assume that the deadline for any particular PHEV is given relative to its arrival time. Specifically, the deadline is $\Delta T$ time after the PHEV arrival, where $\Delta T$ is uniformly distributed in the interval $(0, \Gamma)$. The deadlines for different PHEVs are mutually independent and they are also independent of the arrival time and the charging levels.
1) **PHEV Load Profile:** Recall that the charging rate is constant at the level $\psi$. If each PHEV needs the same amount of power $\Phi$ kWh at arrival, then the total power demand for the arrived PHEVs can be calculated as

$$\Theta(t) = \psi \cdot N_c(t),$$

(2)

where $N_c(t)$ denotes the number of PHEVs that are charging at time $t$, and is given by

$$N_c(t) = \mathcal{N}(\min(t, T_1)) - \mathcal{N}(\max(0, t - \frac{\Phi}{\psi})).$$

(3)

The random process $\Theta(t)$ can be characterized as follows. Consider a particular time instant $t$. Since $\mathcal{N}(t)$ is a Poisson process, $N_c(t)$ should have the same statistical information as $\mathcal{N}(\min(t, \frac{\Phi}{\psi}, T_1 - (t - \frac{\Phi}{\psi})))$. In other words, $N_c(t)$ is a Poisson random variable with the mean given by

$$E[N_c(t)] = \min(t, \frac{\Phi}{\psi}, T_1 - (t - \frac{\Phi}{\psi}))) \lambda.$$  

(4)

If the individual power demand for all the PHEVs is uniformly distributed in the range of $[\frac{\Phi}{2}, \Phi]$, then the mean of the Poisson random variable $N_c(t)$ can be calculated as

$$E[N_c(t)] = \begin{cases} t\lambda & \text{if } t \leq \frac{\Phi}{2\psi} \\ (2t - \frac{\Phi}{2\psi} - \frac{\psi}{\psi} t^2)\lambda & \text{if } \frac{\Phi}{2\psi} < t \leq \frac{\Phi}{\psi} \\ \frac{3\Phi}{4\psi} \lambda & \text{if } \frac{\Phi}{\psi} < t \leq T_1 \end{cases}$$

(5)

If on the other hand the individual power demand for all PHEVs is exponentially distributed with mean $\mu$, then this becomes the standard $M/M/\infty$ model in queueing theory [16]. The distribution of $N_c(t)$ is also Poisson and the mean of the Poisson random variable is given by $\lambda/\mu$.

**B. Limit on PHEVs Charged Simultaneously**

If the service provider can only charge a limited number of PHEVs at any given time (possibly to avoid overload on the distribution system), a queue is formed whenever the power demand for charging exceeds a certain limit. In other words, newly arrived PHEVs will join a queue if charging is not immediately available.

The power constraint necessitates a balance between the utilization level of the power resources and the customer perceived quality of service. As such we define a utilization factor $\rho$ as

$$\rho = \frac{\lambda \cdot E[\gamma_i]}{N_p},$$

(6)

where $N_p$ is the number of PHEVs that can be charged simultaneously, $\gamma_i$ is the time required to fully charge the $i$-th PHEV, and $\lambda$ is the normalized rate defined by $\lambda = \lambda T_1 / T_2$ for the office scenario, and $\lambda = \lambda$ for the mall scenario. The metric used in this paper for the customer perceived quality of service is the average number of PHEVs missing deadlines. On one hand, service providers would prefer $\rho$ to be large such that the required power resource limit is small. On the other hand, a larger $\rho$ will lead to larger number (in average) of PHEVs missing deadlines. Therefore, service providers should choose a proper scheduling strategy according to the PHEV arrival model and operate in a regime where quality and efficiency are balanced.

**C. Scheduling Strategies**

We now proceed to evaluate various scheduling strategies and seek the optimal scheduling strategies for both the office and mall scenarios. For both scenarios, we first consider a special case where there is no cooperation among the charging meters and the PHEVs are assigned to the charging meters randomly and uniformly. Once assigned to a particular charging meter, the PHEV will stay there till it is fully charged. In this case, the arrival for each charging meter is also a Poisson process with the rate $\lambda/N_p$. Then we consider the case when the centralized controller can schedule the charging among all meters in a cooperative way. Charging strategies can also be classified as preemptive, in which a charging job can be split in any number of nonconsecutive time slots, and non-preemptive, in which the charging of any PHEV must happen in a continuous time window. We consider both classes of strategies.

1) **Office Scenario:** When there is no cooperation among the charging meters, the entire system is simply an extension of the case when only one charging meter is present (and the arrival rate is suitably reduced). The performance for a single charging meter where only one PHEV is allowed to be charged at any time can be evaluated as follows.

**Theorem 3.1:** Assume that each PHEV has a power demand that is randomly distributed in $[0, \Omega]$ ($\Omega > 0$). For the single charging meter case in the office scenario, the (preemptive) Shortest Charging-time First Serve (SCFS) scheduling strategy minimizes the number of missed charging deadlines.

**Proof:** At any time instant $t$ ($t < T_2$), the charging strategies need to decide which job gets served first. The SCFS strategy chooses the one that requires the shortest service time. The job schedule with any strategy can be treated as a permutation of the order in which the jobs get served. We show that exchanging the order of any two jobs in the SCFS scheduling strategy cannot reduce the number of PHEVs missing deadlines.

Consider any two charging jobs $\tau_i$ and $\tau_j$ in the SCFS scheduling strategy. The job $\tau_i$ is performed before $\tau_j$. Let the required charging time for them be $\gamma_i$ and $\gamma_j$ respectively. By definition of SCFS algorithm, $\gamma_i < \gamma_j$.

If both jobs succeed or both fail, then the number of PHEVs missing charging deadlines remains the same after exchanging the order of two jobs. If only $\tau_j$ fails in the SCFS scheduling strategy, then the number of PHEVs missing charging deadlines increases or remains the same after exchanging the order of two jobs. Therefore, when the schedule (order) for all other jobs is fixed, the number of PHEVs missing charging deadlines can only increase or remain the same if we change the scheduling strategy by exchanging the order of the two jobs.

**Remark:** This result holds irrespective of the arrival rate of the Poisson process and the distribution of the charging time.
The proofs of Theorem 3.4 and Corollary 3.5 are omitted in this paper because of lack of space. Note that both Theorem 3.4 and Corollary 3.5 hold with the assumption of the exponential service time.

Now we extend the above results to the case of multiple charging meters with cooperation. Under the assumption that a job must be completed whether its deadline expires or not, and with a different optimality metric, [18] shows that the non-preemptive EDF strategy is optimal among the non-preemptive strategies. However, we have not been able to obtain analytical results of optimality without that assumption or if the optimality metric is to minimize the probability of missing deadlines as in this paper. Numerical evaluation of several typical scheduling strategies is carried out in the next section.

IV. NUMERICAL EVALUATION

This section considers specific cases to evaluate the performance of various scheduling strategies for PHEV charging for the case of multiple charging meters. Monte Carlo simulations are carried out in Matlab for both the office scenario and the mall scenario. The time scale is discretized with a step of 5 minutes and decisions for the scheduling strategies are taken at the beginning of every step.

A. Office Scenario

In the office scenario, employees arrive at their corporate parking plaza daily between 7am and 10am according to a Poisson process with mean number of arrivals per day as 1000. The charging level required by each PHEV is uniformly distributed between 0 and 8kWh, and charging is at a constant rate of $\psi = 4$ kW. The fixed deadline for all the PHEVs is set at 5pm.

Four scheduling strategies are considered for the office scenario.

- First Come First Serve (FCFS): Newly entering PHEVs join the tail of a queue. The scheduler selects the PHEV at the head of the queue for charging whenever a charging station becomes available. This can also be considered to be the unscheduled strategy.
- Shortest Charging-time First Serve (SCFS): Newly entering PHEVs join the tail of a queue. The scheduler chooses the PHEV in the queue with the shortest required service time.
- Longest Charging-time First Serve (LCFS): Similar to SCFS, but the scheduler chooses the PHEV with the longest required service time.
- McNaughtan (McN): The details of this strategy can be found in [17].

Figure 2 illustrates a particular realization of the charging for the unlimited resource scenario, and the coordinated charging scenarios with FCFS and LCFS strategies. The peak load of the uncoordinated charging (the unlimited resource) scenario is close to 1500kW, which is far above the peak level (500kW) of the coordinated charging scenarios. Figure 3 compares the performance of the four scheduling strategies for the case
Fig. 2. A particular realization of coordinated charging and noncoordinated charging ($\rho = 0.8$ for both FCFS and LCFS)

Fig. 3. Performance comparison among four scheduling strategies for the office scenario

B. Mall Scenario

In the mall scenario, customers arrive at the mall parking plaza during 8am and 8pm as a Poisson process with mean as one thousand per day. PHEVs are plugged into the charging meters during this time range. The demand for each PHEV and the charging rate follow the same setting as in the office scenario. Each PHEV has a unique deadline uniformly distributed in the range of 0 and 2 hours after the arrival of the PHEV. Note that whether a PHEV is charged or not, it is assumed to leave the system at the completion of its deadline.

We also compare four scheduling strategies for the mall scenario. Other than the FCFS and LCFS strategies, we also consider both non-preemptive and preemptive earliest deadline first (EDF) strategies to arrange PHEVs on the basis of their deadlines. The EDF strategies give preference to the jobs which have the earliest deadline. Figure 4 shows the performance of the four scheduling strategies. The preemptive EDF strategy outperforms the others when the utilization factor is smaller than 1.15. To maintain good quality of service, the system needs to operate with low probability of missing deadlines. If the service provider sets the probability of missing deadlines to be $10^{-6}$, the preemptive and non-preemptive EDF strategies can handle utilization factors around 1 and 0.97, while the FCFS and SCFS strategies only support utilization factors around 0.82 and 0.73. Therefore, the preemptive EDF strategy has the best capability of balancing quality of service and utilization level among the four strategies.

V. CONCLUSIONS AND FUTURE WORK

This paper considered the problem of scheduling PHEV charging in shared commercial parking lots when the arrival of the PHEVs is stochastic and consumers have individual deadlines. When there is no cooperation among different charging meters, this paper provides and analyzes optimal scheduling strategies which minimize the number of missed deadlines. For multiple charging meters with cooperation, this paper numerically evaluates the performance of several typical strategies and provides suggestions for service providers on what regime the system should be operated in to balance system impacts, profit and quality of service.

This paper has only considered scheduling of the time for charging different PHEVs with a constant charging rate. Future work will consider scheduling both the time and the charging rate (different levels of charging rates) which introduces another degree of freedom and potential performance improvement. Future work can also incorporate the fact that the realistic physical charging rate is usually not constant and varies with time and consider scheduling with a discretized model with several levels of rates which approximates the realistic charging rate model.
REFERENCES


